RATIONAL NUMBER

A Professional Development Seminar

presented by

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RMIT University

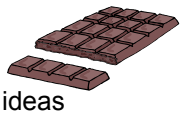
Use the cards below to make two fractions that when added together are as close as possible to one.

$$\frac{\square}{\square} + \frac{\square}{\square}$$

1	3	4	5	7

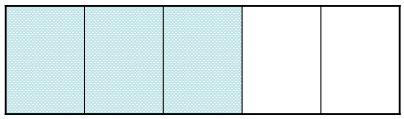
Justify your choice.

OVERVIEW:



- Intuitive, informal fraction ideas
- Partitioning – the missing link in formalising and extending fraction ideas
- Introducing decimal fractions – dealing with a new place-value part
- Extending and applying fraction and decimal ideas

Can you see



3 fifths?

1 and a half?

5 thirds?

$\frac{2}{3}$ of $\frac{3}{5}$ or $\frac{3}{2}$ of $\frac{2}{5}$

What we see depends on what we view as the unit

INTUITIVE, INFORMAL FRACTION IDEAS:

In Prep to Year 3, children need to be exposed to plenty of real-world instances of the 'out of' idea (fraction as *operator*)

- **half** of the apple, half of the class
- a **quarter** of the orange,
- 3 **quarters** of the pizza
- 2 **thirds** of the netball court
- 3 out of 12 eggs are cracked

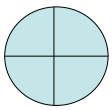
NB: language only, **no symbols**

Use **continuous** and **discrete** fraction models to explore language:

Half of the pizza

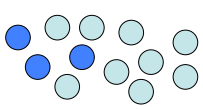
Quarter of an orange

3 **quarter** time



Continuous models are infinitely divisible

Discrete models are collections of wholes



Half a dozen eggs

3 **quarters** of the marbles

1 **third** of the grade

Informally describe and compare:

eg, Is it a big share or a little share? Would you rather have 2 thirds of the pizza or 2 quarters of the pizza? Why?

eg, Explore paper folding, what do you notice as the total number of equal parts increases? What do you notice about the names of the parts?

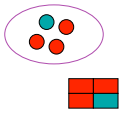
eg, Is it a fair share?

Use non-examples to stress the importance of equal parts

Explore the difference between 'how many' and 'how much'

Recognise: third 3rd

- Fraction words have different meanings, eg, 'third' can mean third in line, the 3rd of April or 1 out of 3 equal parts
- that the *'out of'* idea only works for proper fractions and recognised wholes

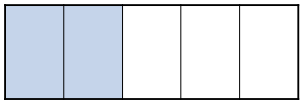


3 out of 4

This idea does not work for improper fractions eg, "10 out of 3" is meaningless!

Recognise also:

- what is involved in working with fraction diagrams, eg, colour to show 2 fifths



Possible to simply count to 2 and colour

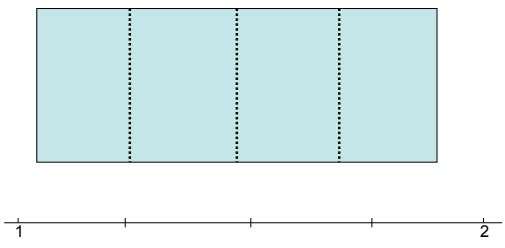
In my view, counting and colouring parts of someone else's model is next to useless - **students need to be actively involved in making and naming their own fraction models**.

PARTITIONING – THE MISSING LINK:

Partitioning (physically dividing wholes or line segments into equal parts) is the key to formalising and extending fraction ideas.

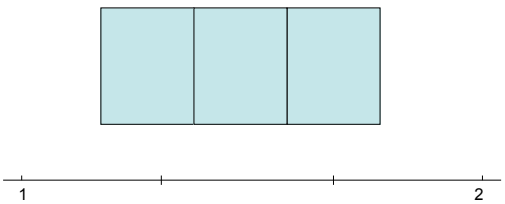
- develop strategies for **halving**, **thirding** and **fifthing** and use to;
- notice key generalisations;
- create diagrams and number lines;
- and to make, name, compare, and rename mixed and proper fractions.

Halving:



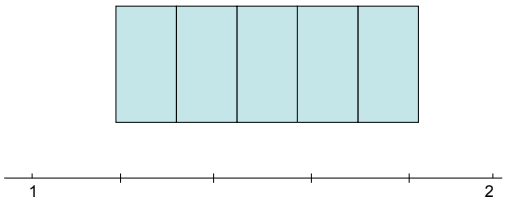
THINK: Halve and halve and halve again ...

Thirding:



THINK: 1 third is less than 1 half … estimate 1 third, leaving room for two more parts of the same size ... halve the remaining part

Fifthing:



THINK: 1 fifth is less than 1 quarter … estimate 1 fifth, leaving room for four more parts of the same size ... halve the remaining part and halve again

Explore strategy combinations:

Eg, What happens if you combine:

(a) **halving** and **thirding**?

(b) **thirding** and **fifthing**?

(c) **halving** and **fifthing**?

What do you notice?

What fractions can be created? How?

Notice that:

The number of parts names the part

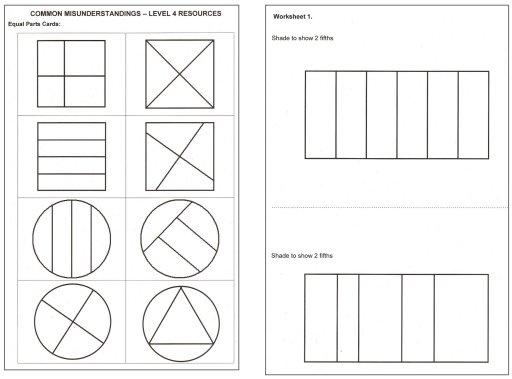
and

as the total number of equal parts increases, the size of each part gets smaller

These are important generalisations that need to be understood to work effectively with fractions

No. parts	Name
1	whole
2	halves
3	thirds
4	quarters
5	fifths
6	sixths
7	sevenths
8	eighths
9	ninths
10	tenths
11	elevenths
12	twelfths
…	…

For example, The Equal Parts Tool (ACM, 4.1)

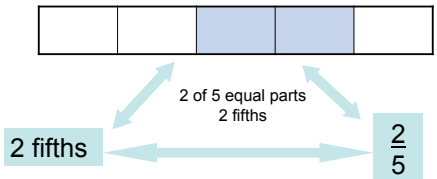


Recording common fractions:

Introduce recording once **key ideas** have been established through practical activities and partitioning, that is:

- equal shares - equal parts Explore non-examples
- as the number of parts increases, the size of each part decreases
- fraction names are related to the total number of parts (denominator idea) This tells **how much**
- the number of parts required tells how many (numerator idea) This tells **how many**

Introduce the fraction symbol:

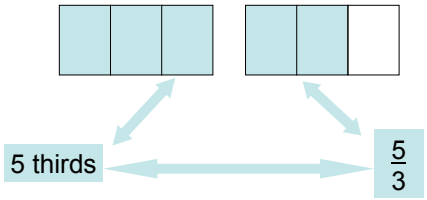


$\frac{2}{5}$ This number tells **how many**

This number names the parts and tells **how much**

Make and name **mixed common fractions** including tenths

For example:



5/3 ← This number tells **how many**

5/3 ← This number names the parts and tells **how much**

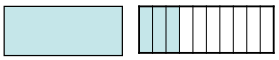
Explore **using region and line models**

INTRODUCING DECIMAL PLACE-VALUE:

Recognise tenths as a new place-value part:

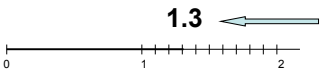
1.Introduce the new unit: **1** one is **10** tenths

2.Make, name and record ones and tenths



1 and 3 tenths

ones	tenths
1	3



The **decimal point** shows where the ones begin (it is not needed in a place-value chart)

3. Consolidate: compare, order, count forwards and backwards in ones and tenths, and rename

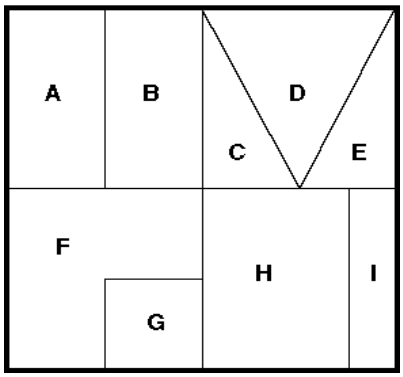
Consolidating fraction knowledge:

- **Make, name,** and **record** using region and line models
- **Compare mixed common fractions and decimals** – which is bigger, which is smaller, why?

  Which is bigger? Why?
  2/3 or 6 tenths ... $1^{1}/_{2}$ or $1^{8}/_{16}$
- **Order common fractions and decimal fractions** on a number line
- **Count forwards and backwards in recognised parts**
- **Rename** in as many different ways as possible.

For example,

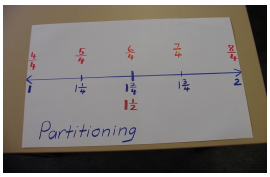
Name the parts

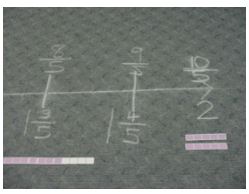


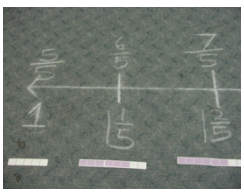
Source: http://www.nottingham.ac.uk/education/MARS/tasks/g8_3/

For example,

Sequence and rename







(Gillian Large, Year 5/6, 2002)

Make a Whole Game

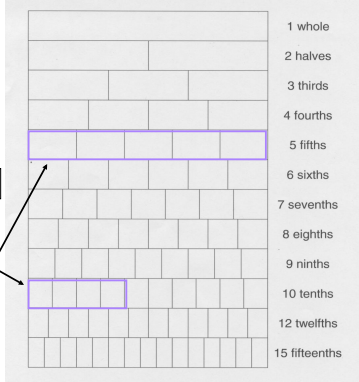
2-6 players

Two Card packs, shuffled and turned upside down.

7 ← a set of numerator cards

and a set of denominator cards → fifths

Students take turns to select one card from each pile and mark their game sheet. Winner is the person with the greatest number of wholes.



EXTENDING FRACTION IDEAS:

By the end of Level 4, students are expected to be able to:

- rename, compare and order fractions with unlike denominators

Requires: partitioning strategies, *fraction as division* idea and *region* idea for multiplication

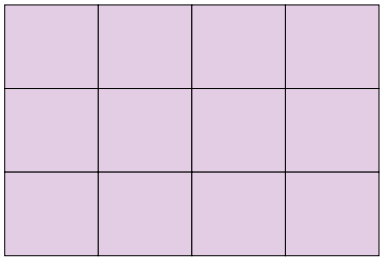
- recognise decimal fractions to thousandths

Requires: partitioning strategies, the place-value idea **1 tenth of these is 1 of those,** and the *for each* idea for multiplication

NOTICE:

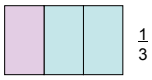
fourths or quarters
(4 parts)

thirds
(3 parts)



thirds by fourths ... twelfths

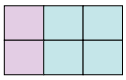
Renaming Common Fractions:



$\frac{1}{3}$

Use paper folding & student generated diagrams to arrive at the generalisation:

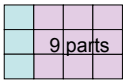
$\frac{2}{6}$



$\frac{3}{4}$

4 parts

$\frac{9}{12}$



12 parts

If the total number of equal parts increase by a certain factor, the number of parts required increase by the same factor

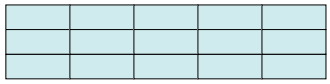
Comparing common fractions:

Which is larger 3 fifths or 2 thirds?

But how do you know? ... **Partition**

fifths

thirds

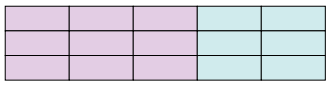


THINK: thirds by fifths ... fifteenths

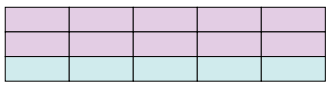
Comparing common fractions:

Which is larger 3 fifths or 2 thirds?

$\frac{3}{5} = \frac{9}{15}$



$\frac{2}{3} = \frac{10}{15}$



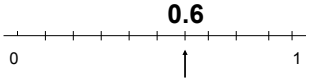
2 thirds > 3 fifths

Linking common and decimal fractions

6 tenths

$\frac{6}{10}$

0.6



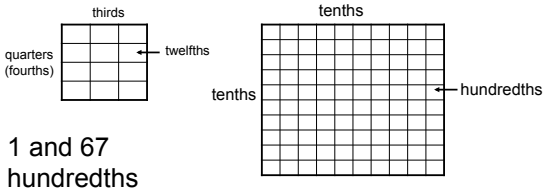
Note that in a place value chart the **decimal point** is not necessary and can be confusing ⇨

Ones	tenths
0	6

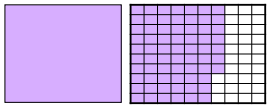
Make and name **ones and tenths** using **region and line models** (e.g., 3 and 4 tenths, 9 and 3 tenths etc)

Linking tenths to hundredths:

Just as thirds by fourths gives twelfths → Tenths by tenths give hundredths



1 and 67 hundredths



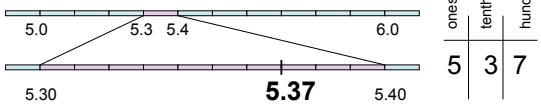
O	t	h
1	6	7

$\frac{167}{100}$ = **1.67**

Extend decimal place-value to hundredths:

Recognise hundredths as a new place-value part:

1. **Introduce the new unit**: 1 tenth is 10 hundredths
2. **Show, name and record** ones, tenths & hundredths via partitioning



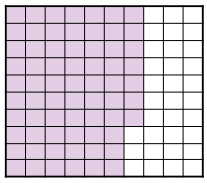
3. **Consolidate**: compare, order, count forwards and backwards, and rename

NB: Money and MAB do NOT work!

Link hundredths to percent:

67 hundredths or 6 tenths and 7 hundredths

$\frac{67}{100}$ 0.67



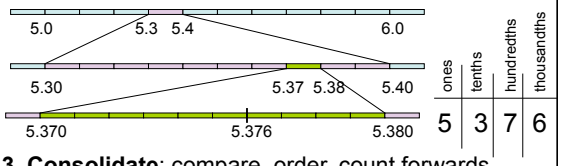
67 percent 67%

Extend by reading to hundredths place, eg, 0.125 can be read as 12 hundredths and 5 thousandths or 12.5 hundredths, that is, 12.5%

Extend decimal place-value to thousandths:

Recognise thousandths as a new place-value part:

1. **Introduce the new unit**: 1 hundredth is 10 thousandths, 1 thousandth is 1 tenth of 1 hundredth via partitioning
2. **Show, name and record** ones, tenths, hundredths and thousandths



3. **Consolidate**: compare, order, count forwards and backwards, and rename

Compare, order and rename decimal fractions:

- **Compare mixed common fractions and decimals** – which is bigger, which is smaller, why?
- **Order common fractions and decimal fractions** on a number line
- **Count forwards and backwards in recognised parts**
- **Rename** in as many different ways as possible

Work with a partner to prepare examples for each.

Some common misconceptions:

- **For fractions:** the numerator and denominator are separate values - the larger the denominator the larger the fraction
- **For decimals:** the more digits the larger the number (e.g., 5.346 said to be larger than 5.6)
- The less digits the larger the number (e.g., 0.4 considered to be larger than 0.52)
- If ones, tens hundreds etc live to the right of 0, then tenths, hundredths etc live to the left of 0 (e.g., 0.612 considered smaller than 0.216)
- Zero does not count (e.g., 3.01 seen to be the same as 3.1)

Target Practice Game:

- 4 ten-sided dice (0-9)
- Target Practice Worksheet per player or team

Players take turns to throw dice, record the numbers thrown, make a number as close to the target as possible (using 3 numbers only), and calculate 'How close'. Winner is player or team with lowest sum

Numbers Thrown	Target	Number Made	How close?
4, 0, 3, 7	**15.6**	30.4	14.8
	3.01		
	7		
		Total	

Game sheet can be adapted to have as many rows as required

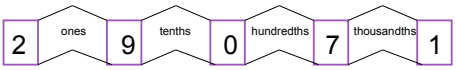
Compare, order and rename decimal fractions:

a) Is 4.57 km longer/shorter than 4.075 km?

b) Order the the long-jump distances: 2.45m, 1.78m, 2.08m, 1.75m, 3.02m, 1.96m and 2.8m

c) 3780 grams, how many kilograms?

d) Express 7¾ % as a decimal



Use Number Expanders to rename decimals